The added term is called the *displacement current density*,

$$\boxed{\vec{J}_d \equiv \epsilon_o \frac{\partial \vec{E}}{\partial t}} \qquad (7.106)$$

It is only called this because it appears in the Ampere's Law equation with the same units and form as a current. It is not a physical current carried by charges.

By construction, $\vec{J}_d$ solves the problem with $\vec{\nabla} \cdot \vec{\nabla} \times \vec{B}$. Let us see how it solves the problem with the integral version of Ampere's Law. The electric field in the capacitor is

$$\vec{E} = \frac{\sigma}{\epsilon_o}\,\widehat{n} = \frac{1}{\epsilon_o}\,\frac{Q}{A}\,\widehat{n} \qquad (7.107)$$

where $\widehat{n}$ is the normal from the positive plate to the negative plate. Therefore, the displacement current is

$$\vec{J}_d = \epsilon_o \frac{\partial \vec{E}}{\partial t} = \frac{1}{A}\,\frac{dQ}{dt}\,\widehat{n} = \frac{\mathrm{I}}{A}\,\widehat{n} \qquad (7.108)$$

The integral form of Ampere's Law with the displacement current is therefore

$$\oint_{\mathcal{C}} d\vec{\ell} \cdot \vec{B} = \mu_o\,\mathrm{I}_{encl} + \mu_o \int_{\mathcal{S}(\mathcal{C})} da\,\widehat{n} \cdot \vec{J}_d \qquad (7.109)$$

If we choose the first surface we discussed earlier, the flat surface in the plane of the contour $\mathcal{C}$, we get the first term but the second term vanishes, which gives $\mu_o\,\mathrm{I}$. If we choose the second surface, the one between the capacitor plates, the first term vanishes but the second term gives $\mu_o\,\mathrm{I}$. Thus, the inconsistency seen earlier has been eliminated.

$x_1 = 2.834$

$y_1 = 2.54 - 0.294$

$y_1 = 2.246$

Now IInd approximation

$f_1 = F(x_1, y_1) = x_1^2 - y_1^2 - 3 = -0.0000$

$g_1 = g(x_1, y_1) = x_1^2 + y_1^2 - 13 = 0.1730$

Now diff. w.r.to x_1, y_1 we get

$\frac{\partial F_1}{\partial x_1} = 2x_1 = 2 \times 2.83 = 5.66 \quad \frac{\partial F_1}{\partial y_1} = -2y_1 = -2 \times 2.2 = -4.46$

$\frac{\partial g_1}{\partial x_1} = \boxed{2x_1} = 2 \times 2.83 = 4.48 \quad \frac{\partial g}{\partial y_1} = 2y_1 = 5.66$

Now replacing these values in cond^n of convergence we get

$h \frac{\partial F_1}{\partial x_1} + K \frac{\partial F}{\partial y_1} = -F(x_1)$

$h \frac{\partial g_1}{\partial x_1} + K \frac{\partial g_1}{\partial y_1} = -g(x_1)$

$h \times 5.66 + K \times 4.48 = 0.0000425 \times 5.66$

$h \times 4.48 + 5.66K = -0.1730 \times 4.48$

$2h(5.66)$

$h = -0.01521, \quad k = -0.01919$

Now the IInd approximation can be written

$x_2 = x_1 + h = 2.83 + 0.015 = 2.815$

$y_2 = y_1 + k = 2.246 - 0.019 = 2.22$

Ques $x^2 - y^2 = 4$ & $x^2 + y^2 = 16$ $\quad 9xy - 7x^2 = 1$

Ques we have given

$F(x, y) = x^3 - 2x^2 + 2$

& $G(x, y) = 9xy - 7x^2 = 1$

Now diff w.r.to 'x' & 'y'

$\frac{\partial F}{\partial x} = 3x^2 - 4x \quad \frac{\partial F}{\partial y} = 0$

$\frac{\partial G}{\partial x} = 9y - 14x \quad \frac{\partial G}{\partial y} = 9x$

Now the value of x & y for non-linear eq^n then we choose $x = 1$ & $y = 4/9$

the cond^n of ittration will be

$\left|\frac{\partial F}{\partial x}\right| + \left|\frac{\partial F}{\partial y}\right| = 3 \times \frac{1}{4} - 4 \times \frac{1}{2} < 1$

$\left|\frac{\partial G}{\partial x}\right| + \left|\frac{\partial G}{\partial y}\right| = 9 \times \frac{4}{9} - 14 \times \frac{1}{2} + 9 \times \frac{1}{2} < 1$

Hence the component of non-algebric eq^n will be

$x_1 = F(x_0, y_0) = \frac{1}{8} - 2 \times \frac{1}{4} + 2$

$= \frac{1 - 4 + 16}{8} = 13/8 = 1.6$

$y_1 = G(x_0, y_0) = 9 \times \frac{1}{2} \times \frac{4}{9} - 7 \times \frac{1}{4} = 2 - \frac{7}{4} = 0.25$

$$-g(x_0) = h\frac{\partial g}{\partial x_0} + k\frac{\partial g}{\partial y_0} \quad \text{— (4)}$$

Now replacing the value of $f(x_0)$ & $g(x_0)$, $\partial f/\partial x_0$, $\partial f/\partial y_0$, $\partial g/\partial x_0$, $\partial g/\partial y_0$ we get -

$$+4 = h(5.6) + \cancel{k(-5.6)}$$

$$-0.32 = h(5.6) + k(5.6)$$

$$2h(5.6) = 3.68 \Rightarrow h = 0.328$$

Hence $k = 0.32 \times 5.6 - 4$
$= -2.1$

Hence Ist approximation can be written as

$$x_1 = 2.8 + 0.32 = 3.12$$

$$y_1 = 2.8 - 2.1 = 0.7$$

Now IInd approximation

$$f_1 = f(x_1, y_1) = x_1^2 - y_1^2 - 4 = (3.12)^2 - (0.7)^2 - 4$$

$$= 9.73 - 0.49 - 4 = 5.24$$

$$g_1 = g(x_1, y_1) = x_1^2 + y_1^2 - 16$$

$$= (3.12)^2 + (0.7)^2 - 16$$

$$= -5.77$$

Now diff w.r.t 'x' & 'y' we get

$$\frac{\partial f_1}{\partial x_1} = 2x_1 = 2 \times 3.1 = 6.24 \qquad \frac{\partial f_1}{\partial y_1} = -2y_1 = -2 \times 0.7 = -1.4$$

$$\frac{\partial g_1}{\partial x_1} = 2x_1 = 6.24 \qquad \frac{\partial g_1}{\partial y_1} = 2y_1 = 1.4$$

Now replacing these value in condn of convergence, we get

$$-f(x_1) = h\frac{\partial f_1}{\partial x_1} + k\frac{\partial f_1}{\partial y_1}$$

$$-g(x_1) = h\frac{\partial g_1}{\partial x_1} + k\frac{\partial g_1}{\partial y_1}$$

$$-5.24 = h(6.24) + \cancel{k(1.4)}$$

$$+5.77 = h(6.24) + \cancel{k(1.4)}$$

$$2h(6.24) = 0.53$$

$$h = \frac{0.53}{2 \times 6.24} = 0.0424$$

$$k = 0.0424 \times 6.24 + 5.24 = 5.50$$

Now the IInd approximn can be written a

$$x_2 = 3.12 + 0.042 = 3.16$$

$$y_2 = 0.7 + 5.50 = 6.2$$

Soln (2) $x^2 + y = 11$ — (1) $\quad y^2 + x = 7$ — (2)

The component of approximation is calcul by replacing $x = y$. Then

$$x^2 + x - 11 = 0$$

$$x = \frac{-1 \pm \sqrt{1+44}}{2} = \frac{-1 \pm 6.7}{2} = 5$$

$$x = -1 + 6.7/2 = 5.7/2 = 2.85$$

(32)

$\rightarrow x_2 = F(x_1, y_1) = (1.6)^3 - 2(1.6)^2 + 2$
$= 4.09 - 3.12 + 2 = 2.97$

$\rightarrow y_2 = G(x_1, y_1) = 9 \times (1.6)(0.25) - 7 \times (1.6)^2$
$= 3.6 - 17.92$
$= -14.32$

Quest $x^2 + y = 11$, $y^2 + x = 7$

Sol^n we have $F(x, y) = 7 - y^2$
$G(x, y) = 11 - x^2$

Now diff w.r.to x & y

$\frac{\partial F}{\partial x} = 0$, $\frac{\partial F}{\partial y} = -2y$

$\frac{\partial G}{\partial x} = -2x$ $\frac{\partial G}{\partial y} = 0$

Now the value of x & y for non-linear eq^n then we choose $y = \sqrt{6} = 2.4$ & $y = \sqrt{10} = 3.23$

Then component of x_0, y_0 will be 1.615 & 1.2 then cond^n of itteration will be

$\left|\frac{\partial F}{\partial x}\right| + \left|\frac{\partial F}{\partial y}\right| = -2 \times 1.21 < 0$

$\left|\frac{\partial G}{\partial x}\right| + \left|\frac{\partial G}{\partial y}\right| = -2 \times 1.615 < 0$

Hence the component of non-algebric eq^n will be

$x_1 = F(x_0, y_0) = 7 - (1.2)^2 = 7 - 1.44$
$= 5.56$

$y_1 = G(x_0, y_0) = 11 - (1.6)^2 = 11 - 2.56$

$x_2 = F(x_1, y_1) = 7 - (8.4)^2 = -63.56$

$y_2 = G(x_1, y_1)$

~~$y_2 = G(x_1, y_1)$~~ $= 11 - (5.9)^2 = -21.49$

$\rightarrow x_3 = F(x_2, y_2) = 7 - (21.49)^2 = -450.96$

$y_3 = G(x_2, y_2) = 11 - (63.56)^2 = -4028.87$

Q. $x^2 - y^2 = 4$ — (1) & $x^2 + y^2 = 16$ — (2)

Sol^n The component of approximation is calculate by replacing $x = y$ Then

$x_0 = y_0 = 2.8$

Then $f(x_0) = (2.8)^2 - (2.8)^2 - 4 = -4$
$g(x_0) = 2 \times (2.8)^2 - 16 = 0.32$

Then Jacobian can be calculated by diff (1) & (2) w.r.to 'x' & 'y' we get

$\frac{\partial F}{\partial x} = 2x$, $\frac{\partial F}{\partial y} = -2y$

$\frac{\partial g}{\partial x} = 2x$ $\frac{\partial g}{\partial y} = 2y$

Now $J(F, g) = \begin{vmatrix} 2x & -2y \\ 2x & 2y \end{vmatrix}$

$= 4xy - (-4xy)$
$= 8xy = 8 \times 2.8 \times 2.8$
$= 62.72 \neq 0$

Then we know that

$-f(x_0) = h\frac{\partial f}{\partial x_0} + k\frac{\partial f}{\partial y_0}$ —(3)